1 Consider the following action (Minkowski metric) that has been modified by the addition of a Gauge fixing term:

$$S = \int d^4x \left[-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2\xi} (\partial_\mu A^\mu)^2 \right] \qquad \alpha \in \mathbb{R}$$

a Get the classical equations of motion for $A_\mu(x)$

b Obtain the equations of motion in momentum space for $\tilde{A}_\mu(k)$

c Write the equations of motion separately for longitudinal, timelike and transverse modes. Show that these equations imply a relation between the longitudinal and timelike modes, while the transverse ones remain free.